JOURNAL OF APPLIED POLYMER SCIENCE VOL. 19, PP. 375-390 (1975)

Bicomponent Flow of Molten Polymers in Annular Dies of Extruders

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Synopsis

Presented are the approximate and exact solutions of the problem of a combined flow of two non-Newtonian fluids having rheological properties described by a power law though a slit channel. This problem is encountered in the design of calibrating dies for extrusion of bicomponent films. The calculation results have been applied to the description of operation of two extruders feeding the same die in the production of bicomponent films from two polymers pertaining to the same class and having different viscosities at a predetermined ratio of layer thicknesses. The calculation values obtained have shown to be in a good agreement with experimental results for co-extrusion of polypropylene grades having different melt indices.

INTRODUCTION

So-called bicomponent films formed by two layers of different materials bonded to each other have been extensively used in polymer technology for the last few years. An intimate contact of the layers is ensured either by welding thereof during coextrusion of polymer melts or by introducing a thin intermediate adhesive layer.

Such films are quite promising for packing purposes and for the production of filaments. In this latter case, various shrinkage properties of the components are of importance, whereby it is possible to produce filaments with a high degree of crimpiness. Therefore, as components, not only different materials but various grades of the same polymer differing in molecular weight and molecular-weight distribution can be used.

One basic equipment scheme for the production of bicomponent films involves the use of a slit calibrating die. Such die is fed with melts from two extruders, both melts being combined only in a forming (calibrating) slit, wherein a stratified flow is obtained.

The problem of describing a process for the production of a bicomponent film resides in the analysis of a stratified flow of two fluids in an extruder die and in matching operating parameters of individual units of a coextrusion plant. The present paper deals with the consideration of this problem.

The main results of prior investigations of the bicomponent extrusion process are described in detail by White et al.[1] Their paper contains also a quite complete list of references relevant to the problem.

THEORY

Newtonian Fluids

The problem under consideration pertains to the class of problems concerning stratified flows of fluids. Some aspects of this problem are discussed by White et al.[1] who, however, presented a final solution only for a combined flow of two Newtonian fluids in a flat channel.

It is an essential precondition for the problem under consideration that both constituent layers have thickness values negligibly small as compared to the annular slit diameter, wherefore the channel curvature may be neglected and the flow may be assumed as being planar and one-dimensional, as shown in Figure 1, where fluid 1 occupies the height from $y = \Delta$ to $y = H$, and fluid 2, from $y = 0$ to $y = \Delta$.

Pressure in each point along the channel length, including the initial cross section, is the same for both fluids. This assumption has also been confirmed by direct measurements performed by Yu and Han.[2]

The velocity distribution over the channel height for a bicomponent flow of two Newtonian fluids with viscosities η_1 and η_2, respectively, is expressed as follows[1]:

$$\left.\begin{aligned} v_1(y) &= \frac{P}{2\eta_1}\left[H^2 - y^2 - 2\Delta(H - y) - \frac{\tau_r}{\eta_1}(H - y)\right] \\ v_2(y) &= \frac{P}{2\eta_2}(2\Delta y - y^2) + \frac{\tau_r}{\eta_2}y \end{aligned}\right\} \quad (1)$$

where the shear stress τ_r at the interface of the layers is

$$\tau_r = \frac{P}{2}\,\frac{(H - \Delta)^2\eta_2 - \Delta^2\eta_1}{(H - \Delta)\eta_2 + \Delta\eta_1} \quad (2)$$

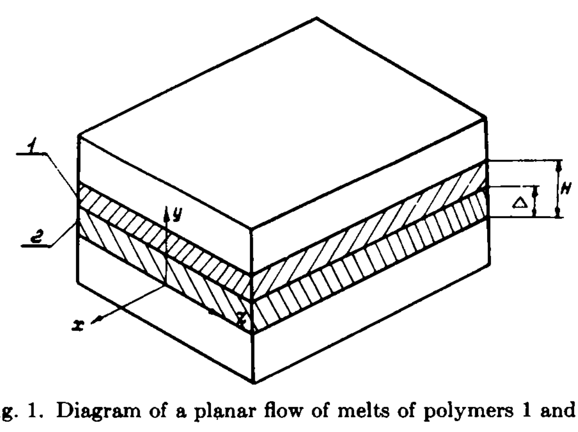

Fig. 1. Diagram of a planar flow of melts of polymers 1 and 2.

and P is the pressure gradient. The boundary condition $v_1(\Delta) = v_2(\Delta)$ is herewith fulfilled. The expression for $v(\Delta)$ given in ref. 1 is likely to be erroneous, however, since its correct form is

$$v(\Delta) = \frac{P}{2\eta_2}\Delta^2 + \frac{\Delta}{\eta_2}\tau_r.$$

In order to ensure the required ratio between thickness of the constituting layers, it is necessary to maintain a predetermined ratio between flow rates $q = Q_2/Q_1$ which, in general, is not equal to the ratio of flow thicknesses in the channel $k = \Delta/(H - \Delta)$, since profile patterns of flow speeds of the two fluids may be different. The thickness ratio of layers of the film being produced is determined by the value of q, not k.

Volume flow rates per unit of channel length are calculated by a conventional method:

$$Q_1 = \int_\Delta^H v_1(y)dy \quad \text{and} \quad Q_2 = \int_0^\Delta v_2(y)dy.$$

Substitution of formula (2) for τ_r and corresponding calculations result in the following relationship between the process parameters:

$$q = \frac{k^2}{\xi}\,\frac{\xi(4k + 3) + k^2}{\xi + k(4 + 3k)} \tag{3}$$

wherein $k = \Delta/(H - \Delta)$; $\xi = \eta_2/\eta_1$; and $q = Q_2/Q_1$. Formula (3) is graphically represented in Figure 2 as a function $q = f(k)$ for different values of the parameter ξ.

Of practical importance is the case where the thicknesses of the constituent layers of the film are equal. In this case $q = 1$, and a simple equation for the function $\xi(k)$ is obtained:

$$k^4 + 4k^3\xi - 4k\xi - \xi^2 = 0. \tag{4}$$

Therefore, for a given q value, a specified relationship is fulfilled between the ratio of the layer thicknesses and the ratio of viscosities of the secltected materials.

Non-Newtonian Fluids. Calculation by Apparent Viscosity

Since real melts comprise non-Newtonian fluids, the practical implementation of the above-obtained results becomes rather difficult. The main difficulty resides in the selection of an appropriate ξ value. A possible method of overcoming this difficulty resides in the evaluation of ξ by apparent viscosities of the melts being coextruded. These values should be consistent with averaged flow conditions in the die. Thus, for the second layer (Fig. 1), the rate gradient according to formula (1) is expressed as

$$\left(\frac{dv}{dy}\right)_2 = \frac{P}{\eta_2}(\Delta - y) + \frac{\tau_r}{\eta_2}$$

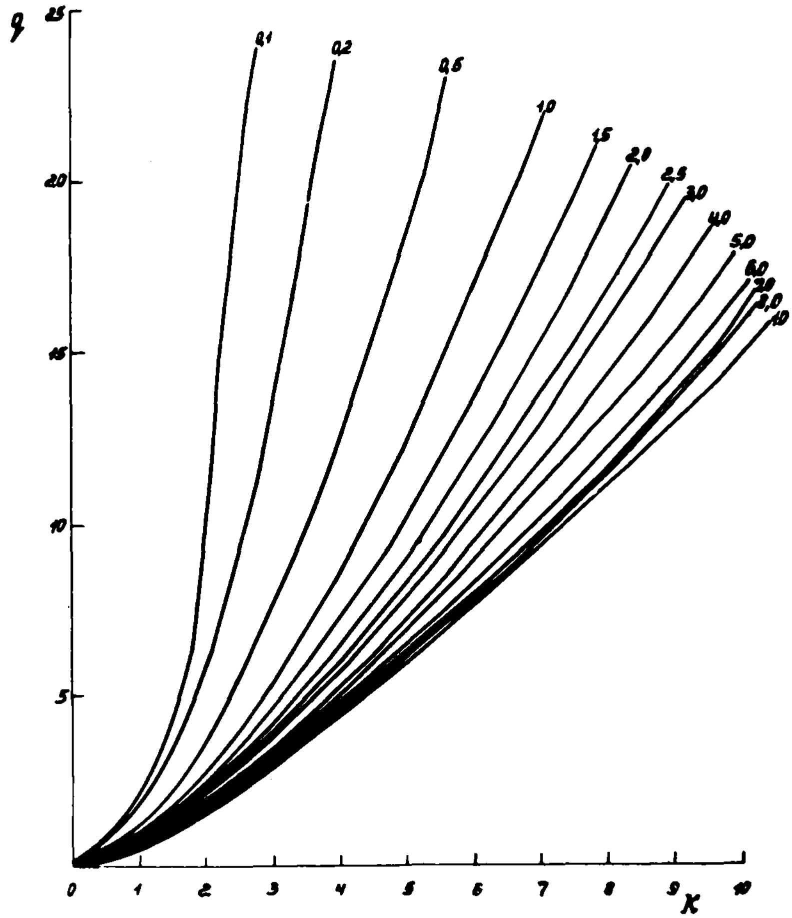

Fig. 2. Graphs of functions of q vs. k for various values of ξ shown by each curve.

which makes it possible to calculate an "average" shear rate ($\dot{\gamma}_{av.}$):

$$(\dot{\gamma}_{av.})_2 = \frac{1}{2}\left[\left(\frac{dv}{dy}\right)_{max} + \left(\frac{dv}{dy}\right)_{min}\right] = \frac{1}{2\eta_2}(\Delta P + 2\tau_\tau)$$

Volume flow rate (per unit of the channel width) in this layer is equal to

$$Q_2 = \frac{P\Delta^3}{3\eta_2} + \frac{\tau_\tau}{2\eta_2}\Delta^2.$$

Comparing the formulae for Q_2 and $(\dot{\gamma}_{av.})_2$, it is seen that $(\dot{\gamma}_{av.})_2$ is approximated as:

$$(\dot{\gamma}_{av.})_2 \approx 3Q_2/2\Delta^2 \tag{5}$$

In a similar way, for the first layer

$$(\dot{\gamma}_{av.})_1 \approx 3Q_1/2(H - \Delta)^2. \tag{6}$$

Expressions (5) and (6) are approximate even for Newtonian fluids. However, from the dimension considerations, it is clear that the apparent shear rate should be proportional to the ratio of a specific volume flow rate Q (cm^2/sec) to the square of the film layer thickness (cm^2), i.e., $\dot{\gamma}_{av.} \sim Q/\Delta^2$. The choice of a numerical factor is of but lesser importance, since involved in further calculations are viscosity ratios and cancellation of the factors which should have entered into accurate formulae for average shear rate and viscosity.

Let us assume that both layers are formed by a polymer of the same nature but having different molecular weights. In this case, viscosimetric properties of the components are characterized by their index melt values I_1 and I_2, respectively comprising conditional values of fluidity at a predetermined shear stress. Within the range of sufficiently high shear-rates and shear stresses, including those at which a melt index is usually measured, a flow curve of thermoplastics is quite well described by a power law:

$$\eta \sim \dot{\gamma}^{-n}$$

where n is a constant referred to as a flow index; for melts of thermoplastics its value is about 0.45–0.7. One may easily see that an apparent viscosity η at a flow with a shear rate γ of a material having a melt index of I is expressed as

$$\eta \sim (\dot{\gamma}^n I^{1-n})^{-1}$$

and then,

$$\xi = \frac{(\dot{\gamma}_{av.})_2}{(\dot{\gamma}_{av.})_1}\left(\frac{I_1}{I_2}\right)^{1-n} = \frac{k^{2n}}{i^{1-n}q^n} \tag{7}$$

where $i = I_2/I_1$ is the ratio of the melt indices of the coextruded components.

Substitution of eq. (7) into eq. (3) after transformations results in

$$q^{2n}k^{2(2-n)}i^{2(1-n)} + q^n k^2(4k+3)i^{1-n} - q^{1+n}k(4+3k)i^{1-n} - qk^{2n} = 0. \tag{8}$$

Consequently, $q(i,k) = 0$ and depends upon n. Expression (8) makes it possible, in a manner similar to that shown in Figure 2, to represent graphically the relationship $q = f(k)$ for different values of the parameter i, which relationship is shown in Figure 3 by solid lines for the values of n equal to 0.4, 0.6, and 0.8, respectively. The use of formula (8) or the corresponding Figure 3 solves the problem of compliance of layer thicknesses in the flow, ratio of components in the final film, and material properties. This solution, however, is approximate due to the use of apparent viscosity values, and its accuracy may be estimated only on the basis of a complete analytical solution of the problem under consideration.

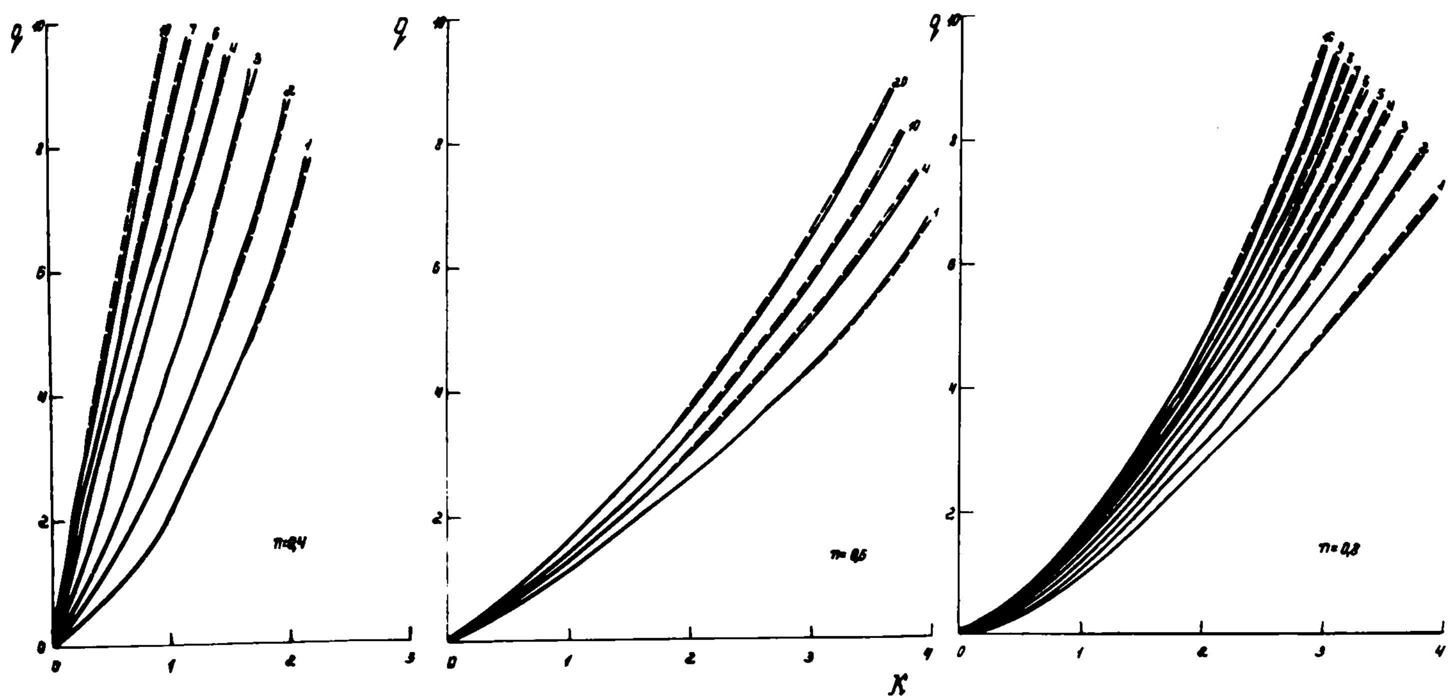

Fig. 3. Graphs of functions $q(k)$ for various i values (shown at each curve). Calculations are performed according to eq. (8) (solid lines) and according to eq. (20) (dotted lines).

Non-Newtonian Flow. Exact Solution

The problem of finding velocity pattern in two layers of stratified flow of non-Newtonian fluids, defining the ratio of volume flow rates, etc., can be solved completely if rheological equations of the fluid state are known. Discussed hereafter are anomalously viscous media possessing no elasticity, wherefore their flow properties are described by certain relationships $\eta_1(|\tau|)$ and $\eta_2(|\tau|)$ which are assumed to be known.

The starting system of equations in the inertialess approximation (which is quite reasonable for highly viscous polymers) is set forth in the following manner:

$$\begin{aligned} \frac{\partial \tau_1}{\partial y} &= -p \qquad \text{at } 0 \le y \le \Delta \\ \frac{\partial \tau_2}{\partial y} &= -p \qquad \text{at } \Delta \le y \le \mathrm{H} \end{aligned} \tag{9}$$

where $p = P/L$ is a pressure drop per unit of the channel length in the stationary flow. Boundary conditions are as follows:

At $y = 0$: $v_2 = 0$; $\tau_2 = \tau_{2,w}$
At $y = H$: $v_1 = 0$; $\tau_1 = \tau_{1,w}$
At $y = \Delta$: $v_1 = v_2 = v_r$; $\tau_1 - \tau_2 = \tau_r$.

Index w denotes stresses at the channel walls, while index r denotes values relevant to the interface of the flows of both fluids (plane of contact of the two fluids).

The solution of eqs. (9) is quite easy:

$$\begin{aligned} \tau_1 &= p(H - y) + \tau_{1,w} \\ \tau_2 &= -py + \tau_{2,w} \end{aligned} \tag{10}$$

and from the boundary condition for the plane $y = \Delta$, formulae connecting $\tau_{1,w}$, $\tau_{2,w}$, and τ_r are obtained:

$$\tau_{2,w} - \tau_{1,w} = pH \tag{11}$$

$$\tau_r = \tau_{1,w} + p(H - \Delta) = \tau_{2,w} - p\Delta \tag{12}$$

Further calculations requiring careful treatment of the signs of rates and stresses are performed on the assumption that the maximum of velocity is in the first layer (at $H > y > \Delta$) and the signs of boundary values for $\dot{\gamma}$ and τ are as follows:

$$\tau_{1,w} < 0; \;\; \tau_{2,w} > 0; \;\; \dot{\gamma}_{1,w} < 0, \text{ and } \dot{\gamma}_{2,w} > 0.$$

Integral condition expressions for velocities have the form

$$v_1(y) = \int_{H-y}^{H} \dot{\gamma}_1 dy = \int_{\tau}^{\tau_{1,w}} \frac{\tau_1 d\tau_1}{p\eta_1(|\tau_1|)} \tag{13a}$$

$$v_2(y) = \int_0^y \dot{\gamma}_2 dy = -\int_{\tau_{2,w}}^{\tau} \frac{\tau_2 d\tau_2}{p\eta_2(|\tau_2|)}. \tag{13b}$$

The boundary condition for velocity on the plane $y = \Delta$ is set forth as follows:

$$\int_{\tau_r}^{\tau_{1,w}} (\tau_1/\eta_1)d\tau_1 = \int_{\tau_r}^{\tau_{2,w}} (\tau_2/(\eta_2)d\tau_2. \tag{14}$$

In order to avoid the necessity to account for the change of the stress sign, eq. (14) should be represented so that it include only positive values:

$$\int_{\tau_r}^{0} \frac{\tau_1 d\tau_1}{\eta_1} + \int_{0}^{\tau_{1,w}} \frac{|\tau_1| d\tau_1}{\eta_1} = \int_{\tau_r}^{\tau_{2,w}} \frac{\tau_2 d\tau_2}{\eta_2}. \tag{15}$$

It is convenient to perform further calculations for particular analytical forms of the functions η_1 and η_2, though it is not difficult to obtain also a general expression for a specific volume output Q which may be found by means of conventional calculation methods for arbitrary functions $\eta(\tau)$ presented analytically or graphically.

Let

$$\eta_1 = \frac{1}{K_1} |\tau_1|^{-m_1}$$

and

$$\eta_2 = \frac{1}{K_2} |\tau_2|^{-m_2}$$

wherein K_1, K_2, m_1, and m_2 are constants, the indices m being simply connected with flow indices n, i.e., $m = n/(1 - n)$; $n = m/(1 + m)$.

Upon integrating eq. (15) with the use of a power law of the flow, we obtain the following equality:

$$\frac{K_1}{m_1 + 2} (|\tau_{1,w}|^{m_1+2}) - \tau_r^{m_1+2}) = \frac{K_2}{m_2 + 2} (\tau_{2,w}^{m_2+2} - \tau_r^{m_2+2}). \tag{16}$$

Formulae (11) and (12) make it possible to exclude τ_r and one of the boundary stress values from eq. (16).

If two materials with the same values of power $m_1 = m_2 = m$ are considered, it means that eq. (16) may be also simplified.

With these considerations in view, eq. (16) may be presented, in dimensionless variables, in any of the following expressions:

$$X_1^{m+2} - (1 - X_1)^{m+2} = -i[(1 - X_1)^{m+2} - (1 + k - X_1)^{m+2}] \tag{17}$$

or

$$\left(\frac{1 + k}{k} - X_2\right)^{m+2} - (X_2 - 1)^{m+2} = i[(X_2 - 1)^{m+2} - X_2^{m+2}] \tag{18}$$

wherein the following designations are used:

$$i = K_2/K_1;\ X_1 = \frac{|\tau_{1,w}|}{p(H - \Delta)};\ X_2 = \frac{\tau_{2,w}}{p\Delta};\ k = \frac{\Delta}{H - \Delta}.$$

Ranges of variation of these values are: $i > 0$; $1 > X_1 > 0$; $X_2 > 1$; $1 > k > 0$; it is not difficult to show that the boundary condition for τ_r results in

$$k = (1 - X_1)/(X_2 - 1). \tag{19}$$

Formulae (17) and (18) connect boundary stresses with the ratio of layer thicknesses in the flow and viscosimetric properties of the coextruded melts.

To solve the problem under consideration, it is necessary to obtain expressions of volume flow rates for each layer Q_1 and Q_2, respectively, the value of the total volume flow rate $Q = Q_1 + Q_2$ and the ratio $q = Q_2/Q_1$ which is equal to the ratio of thicknesses of the constituent layers in the final film.

Calculation of integrals (13a) and (13b) gives the following expressions for the velocity profiles:

$$v_1(y) = \frac{K_1}{p(m+2)} (|\tau_{1,w}|^{m+2} - |\tau_1|^{m+2});\ H > y > \Delta$$

$$v_2(y) = \frac{K_2}{p(m+2)} (\tau_{2,w}^{\ m+2} - \tau_2^{\ m+2});\ \Delta > y > 0$$

and within the range of y values of from Δ to H, the shear stress τ_1 changes its sign.

Calculating Q_1 and Q_2 in a conventional manner, we obtain

$$Q_1 = \frac{K_1}{p^2(m+2)} \left[|\tau_{1,w}|^{m+2}(|\tau_{w,1}| + \tau_r) - \frac{1}{m+3} (|\tau_{1,w}|^{m+3} + \tau_r^{\ m+3}) \right]$$

$$Q_2 = \frac{K_2}{p^2(m+2)} \left[\tau_{2,w}^{\ m+2}(\tau_{2,w} - \tau_r) - \frac{1}{m+3} (\tau_{2,w}^{\ m+3} - \tau_r^{\ m+3}) \right].$$

Further calculations performed by means of eqs. (11) and (12) and introduction of dimensionless variables result in the following final expression for q:

$$q = i\,\frac{1 - X_1}{X_2 - 1}\,\frac{(m+3)\,X_2^{\ m+2} - [X_2^{\ m+3} - (X_2 - 1)^{m+3}]}{(m+3)\,X_1^{\ m+2} - [X_1^{\ m+3} - (1 - X_1)^{m+3}]}. \tag{20}$$

Formulae (17) or (18) and (19) or (20) form a system of three equations the solution of which can give relationships $q(i,k)$ for various meanings of m. Practically, the calculation sequence is defined by the following operations: a value of k is given for a selected value of i; X_1 is found from (17) and $-X_2$ from (19); correct value of X_2 is verified by means of (18), though a reverse approach is possible, i.e., X_2 is found from (18), X_1 from (19), and correctness of X_1 is verified by means of (17); q is obtained from (20). Then, this schedule is repeated for other k values and another i.

A series of relationships $q(i,k)$ for various m (or n) values obtained as a result of the above-described procedure comprises an exact solution of the problem which has been given above, in its approximate form, by eq. (8). The results of calculations performed by means of formulae (17) through (20) are shown graphically in Figure 3 by dotted lines. It is seen from the figure that the values $q(i,k)$ obtained from calculations according to the approximate method are very close to the accurate ones.

Bicomponent Extrusion

The results obtained above establish a correlation between technological process parameters (values of k and i) and performances of the extruders operation (q value).

In principle, two extreme cases are possible: (1) completely independent operation of two extruders, where the resistance of the forming slit in which two flows meet is negligibly small as compared to hydraulic resistance of the remaining channels of the die; and (2) interdependent operation of the extruders feeding a common channel constituting a prevailing portion of the total hydraulic resistance of the die. In the first case, the characteristics of both extruders (dependence $Q(N)$, where N is a screw rotation speed) are extremely "rigid" and do not depend on the value of k (or q). Then, the problem comes down to selection of N_1 and N_2 ensuring the required ratio $q = Q_2/Q_1$ and predetermined total output. The value of k satisfying the predetermined parameters is established automatically, and its estimation is insignificant for the calculation. This case is encountered, for example, in bicomponent annular dies intended for the production of film fibers. In such dies, the forming clearance practically serves only as the point of welding of coextruded layers of the tubular film.

Another extreme case may be roughly exemplified by the conditions occurring in the extrusion of general-purpose films. In this case, one should take into account mutual influence of two operating extruders and match their performances [i.e., relationships $Q(N)$ and $Q(P)$] with the required q value. The theoretical considerations given above and enabling to establish a certain correlation of q, k, and i relate to this case. It should be borne in mind that k is not a parameter of the die design, though it constitutes a parameter which is essential for matching operation of the die and extruders.

Pressure P at the extruder outlet (at the screw end) depends on the die resistance which, in turn (for the arrangement where the main resistance against the flow is due to the calibrating slit), is determined by a thickness of the melt layer δ (δ means either Δ or $H - \Delta$).

It has been shown above that the method of an approximate calculation with respect to apparent viscosities gives satisfactory results for evaluation of integral characteristics of the bicomponent flow. For this reason, it is expedient to apply this method here for the evaluation of hydraulic resistance to the flow in each layer.

Since $\tau \sim P\delta$ and $\tau = \eta\dot{\gamma}$, then

$$P \sim \frac{\eta\dot{\gamma}}{\delta} \sim (\dot{\gamma}^{n}I^{1-n})^{-1}\dot{\gamma}\delta^{-1} \sim (Q/I\delta^{2})^{1-n}\delta^{-1}. \tag{21}$$

Or, introducing a certain proportionality factor ϵ the exact value of which is insignificant, we may write

$$P = \epsilon(Q/I\delta^2)^{1-n}\delta^{-1}. \tag{21a}$$

Then, the performance of each extruder $Q = f(N,P)$ may be presented as $Q = f[N;\ (Q/I\delta^2)^{1-n}\delta^{-1}]$ or $Q = Q(N,\delta)$, though an analytical expression of this function may not be presented in explicit form.

A general approach to the technologic calculation of the bicomponent extrusion resides in a combined analysis of the performances of two extruders $Q_1(N_1,\delta_1)$; $Q_2(N_2,\delta_2)$ and eq. (8) (or the corresponding exact solution of the problem discussed in the preceding paragraph and represented in Fig. 3). This makes it possible to exclude from the consideration the k value which is not defined from the plant arrangement and to obtain, thereby, the relationship between Q_1 and Q_2, and N_1 and N_2, which is the final object of the calculation.

This approach is illustrated in the most descriptive manner by a practically important example of a linear characteristic of an extruder:

$$Q = \alpha N - \beta P \tag{22}$$

where α and β are constants of a direct ("forced") and backward flows, i.e., factors depending on the design of a given extruder.

Substituting eq. (21a) for P into (22), we obtain

$$Q = \alpha N - \beta \frac{\epsilon}{\delta} (Q/I\delta^2)^{1-n}$$

wherefrom

$$(H - \Delta)^{3-2n} = \frac{\beta_1\epsilon}{\alpha_1 N_1 - Q_1}\left(\frac{Q_1}{I_1}\right)^{1-n};\ \Delta^{3-2n} = \frac{\beta_2\epsilon}{\alpha_2 N_2 - Q_2}\left(\frac{Q_2}{I_2}\right)^{1-n}.$$

These formulae make it possible to exclude from consideration an indefinite value of the coefficient ϵ and to connect Q_1 and Q_2 with k. In fact, a member-by-member division results in

$$k^{3-2n} = \frac{\beta_2}{\beta_1}\left(\frac{\alpha_1 N_1 - Q_1}{\alpha_2 N_2 - Q_2}\right)\left(\frac{q}{i}\right)^{1-n}$$

or

$$k^{3-2n} = \frac{\beta_2}{\beta_1}\left[\frac{\alpha_1 N_1(1 - q) - Q}{\alpha_2 N_2(1 - q) - qQ}\right]\left(\frac{q}{i}\right)^{1-t} \tag{23}$$

where $Q = Q_1 + Q_2$ is a total predetermined output.

The thus obtained formula (23) makes it possible, for given values of the output Q and ratio q for the selected polymers (i.e., for a known $i = I_2/I_1$), to find the values of N_1 and N_2 satisfying the above expression. In doing so, the value of k is obtained from eq. (8), so that the solution of this problem comprises, in general, a combination of eq. (8) and formula (23).

If, for some reasons, it is impossible to select values of N_1 and N_2 matching formula (12), it means that either the selected pair of polymers should be

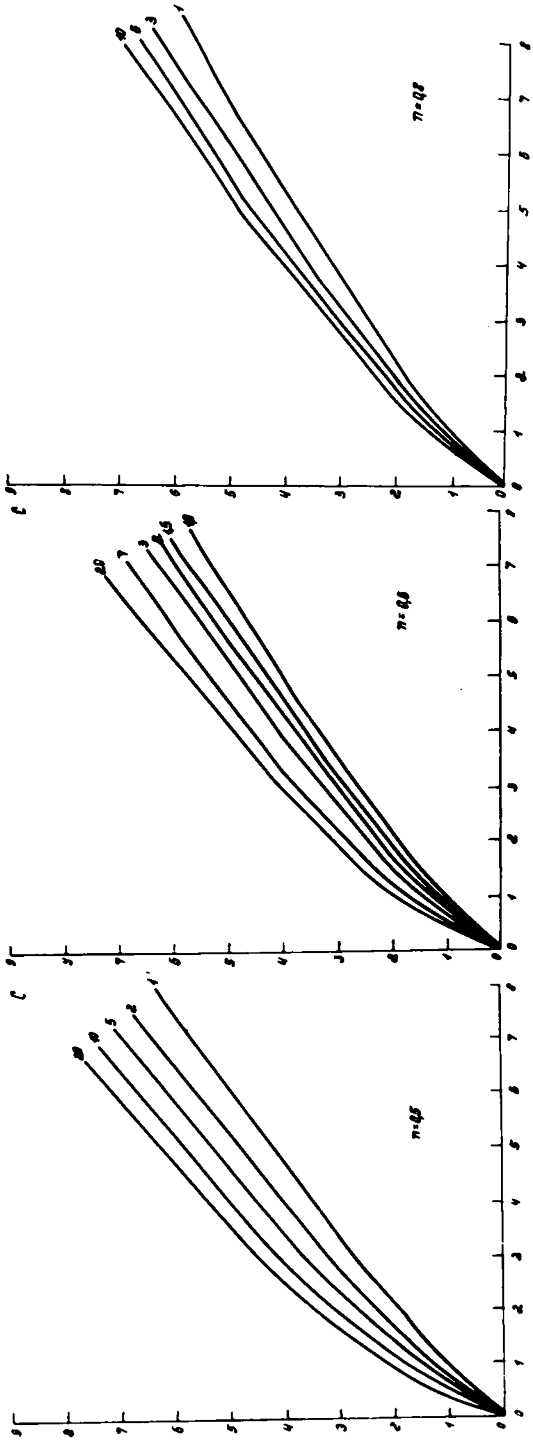

Fig. 4. Functions $C(q)$ for various i values (shown at each curve).

changed, i.e., the i value, or the required output Q, cannot be achieved under any operating conditions. Relationships (8) and (23) may be also used for the selection of a coextruded pair of polymers.

To facilitate concrete calculations, it is advisable that instead of k somewhat different function of i and q be used, i.e.,

$$C(i,q) = k^{3-2n}(i/q)^{1-n}.$$

Since $k(i,q)$ is known from Figure 3, $C(i,q)$ may be easily found. The results of corresponding calculations are presented in Figure 4. Now, the principal calculation formula is written as follows:

$$Q = \frac{[\alpha_1 N_1 - C(\beta_2/\beta_1)\alpha_2 N_2](1 + q)}{1 - C(\beta_1/\beta_2)q} \tag{24}$$

where $C = C(i,q)$ is found from Figure 4.

This formula, in its structure, comprises an expression of the form

$$Q = AN_1 - BN_2 \tag{25}$$

where the meaning of coefficients A and B is evident from (24). The latter relationship comprises the final result of the calculations and may be subjected to an experimental verification. This verification should consist of the following steps: (a) independent finding of parameters of the extruders and values α_1, β_1, α_2, and β_2; (b) determination of C from Figure 4 for the selected pair of the materials (i.e., n and i should be known) and given ratio of the layer thicknesses (i.e., q); (c) calculation of A and B and, from (25), of Q. The thus obtained Q value is compared with the experimentally measured Q values for different values of N_1 and N_2.

EXPERIMENTAL

Verification of the resulting relationships with respect to a technologic coextrusion process was effected in the processing of polypropylene by means of a plant intended for the production of bicomponent films (Reifenhäuser, Federal Republic of Germany).

In the experiments use was made of industrial samples of polypropylene, Soviet grades, having the following characteristics:

Sample	Melt index (at 230°C under 2.16 kgf load)
A	0.55
B	1.85
C	3.85

All the experiments were conducted at a die temperature of 230°C.

Investigation of rheological properties of these polymers had been performed earlier.[3] On the basis of these investigations, it may be assumed that flow curves for polypropylene melts within the range of high shear

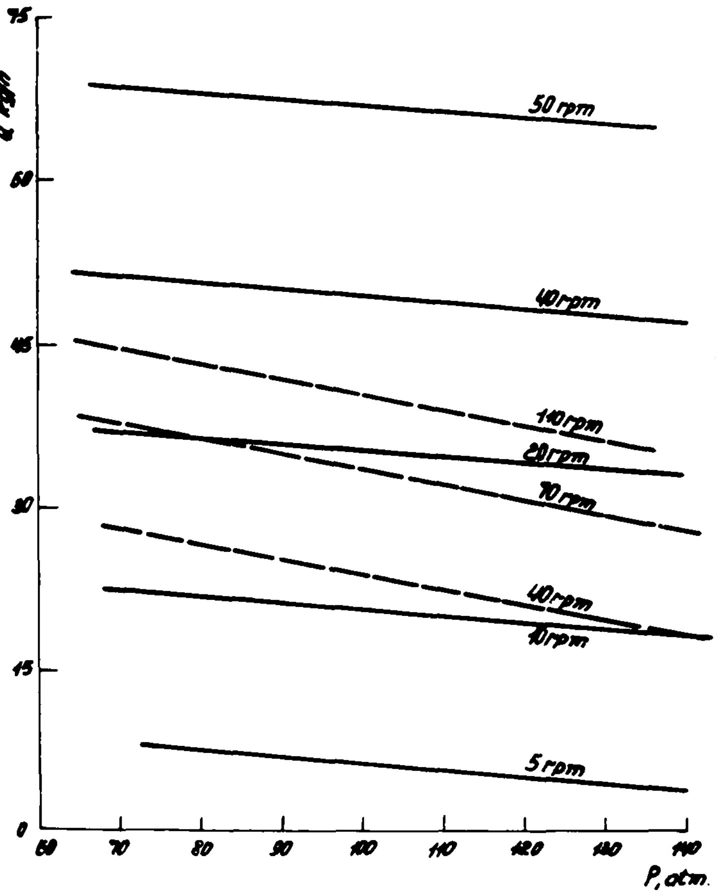

Fig. 5. Functions $Q(P,N)$ for extruders S090 (solid lines) and S060 (dotted lines).

rates are approximated with an altogether satisfactory accuracy by a power law with n of about 0.6.

The plant consisted of two one-screw extruders of Models S090 and S060 and a common "bicomponent" die. Screws with a relative length $L/D = 25{:}1$ were 90 and 60 mm in diameter, respectively. The extruders' performances are shown in Figure 5 in the form of functions $Q(P,N)$.

The treatment of experimentally obtained relationships shown in this figure demonstrated that performances of both extruders may be described by a linear function with the following parameter values: $\alpha_1 = 0.815$, $\beta_1 = 0.125$ for the first extruder and $\alpha_2 = 1.35$, $\beta_2 = 0.075$ for the second.

The experiments were aimed at measuring a net volume flow rate (total) Q and q values for various given screw speeds N_1 and N_2. An error encountered in the measurements of Q was negligibly small, since the Q values used for the comparison with the theoretical results were obtained

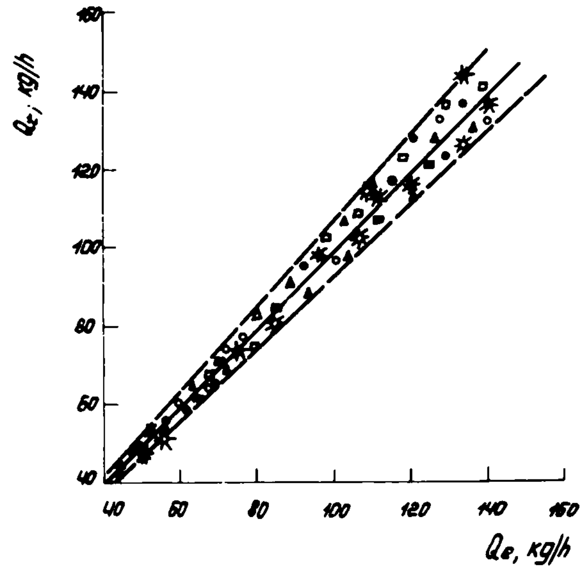

Fig. 6. Comparison of output values of the plant calculated from theory (Q_t) and obtained experimentally (Q_e). Solid line corresponds to the condition of an exact equality $Q_t = Q_e$; dotted line, to a deviation therefrom by $\pm 7\%$. Points represent experimental data at the values of q: 0.3 (○ and ●); 1.0 (△ and ▲); 2.0 (□ and ■); 3.5 (☼ and ✹). Clear labels are results of comparison of Q_e with Q_t values found by an approximate method, while black labels correspond to the exact solution.

by way of "averaging" rather prolonged measurements performed under real production conditions of the plant operation for a period of several hours.

Comparison of the experimental results with the data for Q obtained from the theory (using approximate and accurate formulae) described above is shown in Figure 6. The experimental data were obtained by vaying N_1 from 40 to 100 rpm and N_2 from 10 to 70 rpm. Values of q were varied within the range of about 0.3 to about 3.5.

It is evident that the herein-proposed calculation method makes it possible to obtain Q values with an error of down to $\pm 5\%$ for an accurate solution with the use of formula (20) and to $\pm 7\%$ for an approximate solution based on the use of formula (8). This gives altogether satisfactory compliance with the experiment. The necessity of using the resulting theoretical relationships for the calculation of Q resulting in formula (25) is due to the fact that performances of the extruders (see Fig. 5) are not so rigid that one could try to evaluate the total plant output without taking into account their mutual influence during the operation with the common die as well as the existence of interdependence between P and Q_1 and Q_2.

The authors would like to express their gratitude to A. S. Saveliev for drafting a program and performing all concrete computations from the obtained formulae by means of a computor and V. I. Vlasov for assistance in carrying out the experiments.

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Received June 12, 1974